Generalizing Truth-Functionality

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A common standard for the interpretation of classical propositional logic is set by the functionally complete collection of 2-valued truth-tables. The structure of the free algebra of formulas is faithfully mirrored in the semantics: In each admissible model, each sentential letter freely ranges over the set of truth-values, and to each $n$-ary logical constant there corresponds a convenient $n$-ary operator over those same truth-values. The whole approach is easily generalizable so as to define the class of truth-functional logics, i.e., many-valued logics whose operators can be characterized by truth-tables over some convenient set of truth-values.

A further interesting generalization of the above idea is produced by the so-called non-deterministic truth-tabular semantics ($N$-truth-tables), where each $n$-ary operator is allowed to choose in between a number of possible outputs for each given $n$-uple of inputs. The set of admissible models may thus be enlarged, allowing for the natural adequate interpretation of more generous classes of non-classical logics. Yet another generalization of the same idea is given by the so-called possible-translations semantics with many-valued ingredients (Many-valued PTS), where each model of a logic $L$ is given by an admissible translation of $L$ into an appropriate many-valued logic $L_k$ coupled with a standard many-valued valuation from $L_k$.

In the finite-valued case, logics with truth-tabular or $N$-truth-tabular semantics share interesting meta-properties such as compactness and decidability. Finite-valued PTS are also guaranteed to share those properties as soon as all the corresponding admissible translations are recursively defined. However, several logics known to be uncharacterizable by finite-valued truth-tables can be adequately characterized by finite-valued $N$-truth-tables, and several logics that have no finite-valued $N$-truth-tabular characterization can be characterized by finite-valued PTS.

The present contribution will examine and illustrate the multiple relations between the three above mentioned alternative styles of semantics.

*Research developed also under the scope of the cooperation agreement 24P-22335/04 between CLE-UNICAMP and CLC-UTL, with partial support from EU FEDER and FCT. The author is greatly indebted to Arnon Avron, who kindly agreed to present the contribution during the 2005 ASL Winter Meeting.